

$\mathcal{N} = 2$ SUSY symmetries for a moving charged particle under influence of a magnetic field: Supervariable approach

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Abstract: We exploit the supersymmetric invariant restrictions (SUSYIRs) on the supervariables to derive the nilpotent $\mathcal{N} = 2$ SUSY transformations for the supersymmetric quantum mechanical model of the motion of a charged particle in the X-Y plane (where the magnetic field (B_z) is applied along the Z-direction). The supervariables are defined on a (1, 2)-dimensional supermanifold parametrized by a bosonic “time” variable t and a pair of Grassmannian variables θ and $\bar{\theta}$ (with $\theta^2 = \bar{\theta}^2 = 0$, $\theta\bar{\theta} + \bar{\theta}\theta = 0$). We take the (anti-)chiral supervariables for our purpose so that the nilpotency property of the $\mathcal{N} = 2$ SUSY symmetry transformations could be captured within the framework of supervariable approach. We express the Lagrangian as well as supercharges in terms of the supervariables (that are obtained after the application of the appropriate SUSYIRs) and provide geometrical basis, within the framework of our supervariable approach, for (i) the nilpotency property of the above SUSY transformations (and the corresponding supercharges), and (ii) the SUSY invariance of the Lagrangian.

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1 Introduction

The well-known Becchi-Rouet-Stora-Tyutin (BRST) formalism is one of the mathematically rich and theoretically useful approaches to covariantly quantize the gauge theories where the local gauge symmetries of the original theory are traded with the nilpotent (anti-)BRST symmetries. Two of the abstract mathematical properties associated with the above (anti-)BRST symmetries are the nilpotency and absolute anticommutativity. The geometrical origin and interpretations for the above mentioned nilpotency and anticommutativity properties are provided by the superfield formalism [1-8]. In particular, the Bonora-Tonin (BT) superfield approach [4,5] has been very successful in the context of gauge theories where the horizontality (HC) condition plays a very crucial role. The latter condition leads to the derivation of “quantum” (anti-)BRST symmetries for the gauge and corresponding (anti-)ghost fields which turn out to be nilpotent of order two and absolutely anticommuting *but* it does *not* say anything about matter fields.

In a set of papers [9-13], we have extended the BT-superfield formalism* where, in addition to the HC, we have exploited the gauge invariant restrictions (GIRs) to derive the (anti-)BRST symmetry transformations for the *matter* fields, too, in an *interacting* gauge theory. The symmetries (and their geometrical interpretations) turn out to be consistent with one-another when the HC and GIRs are tapped *together* within the framework of augmented version of BT superfield formalism [9-13]. It has been a long-standing problem to apply the above superfield formalism [1-13] to derive the SUSY transformations for the SUSY systems where the nilpotency property exists but the anticommuting property does *not*. In a very recent set of papers [14,15], however, we have applied the augmented version of superfield/supervariable formalism [9-13] to derive the $\mathcal{N} = 2$ SUSY transformations in a consistent and cogent manner (for the specific $\mathcal{N} = 2$ SUSY quantum mechanical models). We have coined the word *supervariable approach* for our method of derivation of SUSY symmetries (cf. footnote just before (10)).

The supervariable approach [14,15] to derive the SUSY symmetries for the $\mathcal{N} = 2$ quantum mechanical systems is a *novel* approach in the context of SUSY theories. The purpose of our present investigation is to exploit the theoretical tools and techniques of our earlier works on the supervariable approach [14,15] to derive the $\mathcal{N} = 2$ SUSY transformations for the SUSY system of a moving charged particle in the X-Y plane under influence of a magnetic field that is applied along the Z-direction (i.e. perpendicular to the X-Y plane). We express the conserved charges and Lagrangian in the language of supervariables and provide the geometrical interpretations for the SUSY invariance of the Lagrangian as well as the nilpotency of the conserved charges in terms of the translational generators along the Grassmannian directions $(\bar{\theta})\theta$ of the (anti-)chiral super-submanifolds, respectively. These generators are defined on the (1, 1)-dimensional super-submanifolds of the general (1, 2)-dimensional supermanifold on which our *starting* theory is generalized within the framework of supervariable approach.

Our present investigation has been motivated by the following key factors. First and foremost, to put our central ideas [14,15] on a solid-footing, it is essential that we should

*This extended version of the geometrical BT-superfield formalism has been christened as the augmented version of superfield formalism [9-13].

apply the supervariable approach to the models with superpotentials that are completely different from the superpotentials of the $\mathcal{N} = 2$ SUSY free particle, harmonic oscillator (HO) and the generalized version of the HO [16,17]. This is the reason that, in our present investigation, we have taken the SUSY example of the motion of a charged particle under influence of a magnetic field and have demonstrated the utility of our supervariable approach. Second, it has been a long-standing problem to apply some *form* of the superfield approach [1-13] to capture the nilpotency of the SUSY symmetries and provide a geometrical meaning to it. We have accomplished this goal in our present investigation (and in our earlier works [14,15]). Finally, our method of application of supervariable/superfield formalism might turn out to be useful in the context of SUSY gauge theories.

The contents of our present investigation are organized as follows. First of all, we discuss the bare essentials of the $\mathcal{N} = 2$ SUSY transformations for the motion of a charged particle under influence of a magnetic field in Sec. 2. We exploit the virtues of (anti-)chiral supervariables to derive the two nilpotent $\mathcal{N} = 2$ SUSY transformations in Sec. 3. We discuss about the SUSY invariance of the Lagrangian of the theory and nilpotency of the $\mathcal{N} = 2$ SUSY charges within the framework of supervariable formalism, in our Sec. 4. Our Sec. 5 deals with the cohomological aspects of the $\mathcal{N} = 2$ SUSY transformations and corresponding symmetry generators. Finally, in Sec. 6, we make some concluding remarks and point out a few future directions for further investigation.

We provide the logical reasons behind our choice of the (anti-)chiral supervariables in our Appendix A.

2 Preliminaries: $\mathcal{N} = 2$ SUSY symmetries

We begin with the following Lagrangian for the motion of a charged particle (of mass $m = 1$ and charge $e = 1$) in the X-Y plane (see, e.g. [16,17]):

$$L_0 = \frac{1}{2} (\dot{x}^2 + \dot{y}^2) - (\dot{x} A_x + \dot{y} A_y) + i \bar{\psi} \dot{\psi} + B_z \bar{\psi} \psi, \quad (1)$$

where the magnetic field $B_z = \partial_x A_y(x, y) - \partial_y A_x(x, y)$ is in the Z-direction and the whole trajectory of the particle is parametrized by the evolution “time” parameter t . As a consequence, we have the “generalized” instantaneous velocities of the SUSY particle as: $\dot{x} = dx/dt$, $\dot{y} = dy/dt$ and $\dot{\psi} = d\psi/dt$. The instantaneous position variables $x(t)$ and $y(t)$ are bosonic in nature and variables $\psi(t)$ and $\bar{\psi}(t)$ are fermionic ($\psi^2 = \bar{\psi}^2 = 0$, $\psi \bar{\psi} + \bar{\psi} \psi = 0$) at the *classical* level. The X-Y components of the vector potentials (A_x, A_y) have no *explicit* “time” dependence and they are *only* function of the instantaneous position of the particle (i.e. $A_x(x, y)$, $A_y(x, y)$).

It can be readily checked that the starting Lagrangian (1) respects the following $\mathcal{N} = 2$ SUSY transformations s_1 and s_2 [16,17]

$$\begin{aligned} s_1 x &= \psi, & s_1 y &= -i \psi, & s_1 \bar{\psi} &= i (\dot{x} - i \dot{y}), & s_1 \psi &= 0, \\ s_1 A_x &= (\partial_x A_x - i \partial_y A_x) \psi, & s_1 A_y &= (\partial_x A_y - i \partial_y A_y) \psi, \\ s_2 x &= \bar{\psi}, & s_2 y &= i \bar{\psi}, & s_2 \bar{\psi} &= 0, & s_2 \psi &= i (\dot{x} + i \dot{y}), \\ s_2 A_x &= \bar{\psi} (\partial_x A_x + i \partial_y A_x), & s_2 A_y &= \bar{\psi} (\partial_x A_y + i \partial_y A_y), \end{aligned} \quad (2)$$

because the Lagrangian transforms as:

$$\begin{aligned} s_1 L_0 &= -\frac{d}{dt} \left[(A_x - i A_y) \psi \right], \\ s_2 L_0 &= +\frac{d}{dt} \left[\bar{\psi} \{ \dot{x} + i \dot{y} - (A_x + i A_y) \} \right]. \end{aligned} \quad (3)$$

This establishes that the relevant action integral $S = \int dt L_0$ remains invariant under the continuous transformations s_1 and s_2 .

We point out that the above infinitesimal transformations are off-shell nilpotent of order two (i.e. $s_1^2 = s_2^2 = 0$) which establishes their fermionic nature. This is the reason that the above transformations change bosonic variables into fermionic variables and *vice-versa*. Furthermore, we note that the anticommutator of the fermionic transformations s_1 and s_2 leads to a bosonic symmetry transformation (i.e. $s_\omega = \{s_1, s_2\}$), namely;

$$s_\omega \Phi = \dot{\Phi}, \quad \Phi = x(t), y(t), \psi(t), \bar{\psi}(t), A_x(x, y), A_y(x, y), \quad (4)$$

modulo a factor of $(2i)$. In the derivation of the above bosonic symmetry transformations, we have used (for obvious reasons) the following inputs:

$$\begin{aligned} \partial_x \psi(t) &= 0, & \partial_y \psi(t) &= 0, & \partial_x \bar{\psi}(t) &= 0, & \partial_y \bar{\psi}(t) &= 0, \\ \frac{d}{dt} A_x(x, y) &= \dot{x} \partial_x A_x + \dot{y} \partial_y A_x, & \frac{d}{dt} A_y(x, y) &= \dot{x} \partial_x A_y + \dot{y} \partial_y A_y. \end{aligned} \quad (5)$$

Under the above transformations (4), the starting Lagrangian L_0 transforms to a total “time” derivative (of itself) as follows:

$$s_\omega L_0 = \frac{d}{dt} [L_0], \quad (6)$$

which demonstrates the invariance of the action integral $S = \int dt L_0$. It is straightforward to check that s_ω commutes with *both* the fermionic transformations $s_{(1)2}$ (i.e. $[s_\omega, s_1] = 0, [s_\omega, s_2] = 0$).

According to Noether’s theorem, the above continuous transformations lead to the derivation of conserved charges Q_i (with $i = 1, 2, 3$) as

$$\begin{aligned} Q_1 \equiv Q &= \left[(p_x + A_x) - i (p_y + A_y) \right] \psi, & p_x &= \dot{x}, \\ Q_2 \equiv \bar{Q} &= \bar{\psi} \left[(p_x + A_x) + i (p_y + A_y) \right], & p_y &= \dot{y}, \\ Q_3 \equiv Q_\omega &= \left[\frac{(p_x + A_x)^2}{2} + \frac{(p_y + A_y)^2}{2} - B_z \bar{\psi} \psi \right] \equiv H. \end{aligned} \quad (7)$$

The conservation ($\dot{Q}_i = 0$) of the above charges Q_i can be proven directly by using the following Euler-Lagrange equations of motion:

$$\begin{aligned} \dot{\psi} - i B_z \psi &= 0, & \ddot{x} + \dot{y} B_z - (\partial_x B_z) \bar{\psi} \psi &= 0, \\ \dot{\bar{\psi}} + i B_z \bar{\psi} &= 0, & \ddot{y} - \dot{x} B_z - (\partial_y B_z) \bar{\psi} \psi &= 0, \end{aligned} \quad (8)$$

which are derived from the Lagrangian L_0 . The above conserved charges are the generators of the infinitesimal symmetry transformations listed in (2) and (4). This can be explicitly checked by the following general relationship for the generic variable Φ of our present theory, namely;

$$s_r \Phi = \pm i [\Phi, Q_r]_{\pm}, \quad r = 1, 2, \omega, \quad (9)$$

where the (\pm) signs (expressed as the subscripts) on the square bracket correspond to the (anti)commutator for the generic variable $\Phi = x, y, A_x, A_y, \psi, \bar{\psi}$ being (fermionic) bosonic in nature.

3 (Anti-)chiral supervariables: SUSY transformations

To derive the transformations s_1 and its nilpotency, we choose the anti-chiral supervariables (corresponding to *all* the ordinary dynamical variables of the starting Lagrangian L_0) on the (1, 1)-dimensional super-submanifold of the general (1, 2)-dimensional supermanifold on which our present SUSY theory is generalized[†]. In other words, first of all, we generalize the simple variables $(x(t), y(t), \psi(t), \bar{\psi}(t))$ onto the (1, 1)-dimensional anti-chiral super-submanifold as anti-chiral supervariables[‡] (see, e.g. [14,15]):

$$\begin{aligned} x(t) &\longrightarrow X(t, \theta, \bar{\theta})|_{\theta=0} \equiv X(t, \bar{\theta}) = x(t) + \bar{\theta} f_1(t), \\ y(t) &\longrightarrow Y(t, \theta, \bar{\theta})|_{\theta=0} \equiv Y(t, \bar{\theta}) = y(t) + \bar{\theta} f_2(t), \\ \psi(t) &\longrightarrow \Psi(t, \theta, \bar{\theta})|_{\theta=0} \equiv \Psi(t, \bar{\theta}) = \psi(t) + i \bar{\theta} b_1(t), \\ \bar{\psi}(t) &\longrightarrow \bar{\Psi}(t, \theta, \bar{\theta})|_{\theta=0} \equiv \bar{\Psi}(t, \bar{\theta}) = \bar{\psi}(t) + i \bar{\theta} b_2(t), \end{aligned} \quad (10)$$

where the secondary variables (b_1, b_2) and (f_1, f_2) are bosonic and fermionic in nature, respectively. We note that the bosonic (i.e. x, y, b_1, b_2) and fermionic $(\psi, \bar{\psi}, f_1, f_2)$ d.o.f. *do* match on the r.h.s. of the above anti-chiral expansions (cf. (10)) which is one of the key requirements of a SUSY theory.

A decisive feature of the augmented version of BT-superfield formalism [9-13] and our earlier works [14,15] is the requirement that *all* the gauge/SUSY invariant quantities must remain independent of the Grassmannian variables θ and $\bar{\theta}$ when they are generalized onto a specific supermanifold. We observe that such invariant quantities, w.r.t. s_1 , are as follows:

$$\begin{aligned} s_1 [\psi(t)] &= 0, \quad s_1 [x(t) \psi(t)] = 0, \quad s_1 [y(t) \psi(t)] = 0, \quad s_1 [\dot{x}(t) \dot{\psi}(t)] = 0, \\ s_1 [\dot{y}(t) \dot{\psi}(t)] &= 0, \quad s_1 \left[\frac{1}{2} (\dot{x}^2(t) + \dot{y}^2(t)) + i \bar{\psi}(t) \dot{\psi}(t) \right] = 0. \end{aligned} \quad (11)$$

[†]We are theoretically compelled to choose the (anti-)chiral supervariables because the nilpotent $\mathcal{N} = 2$ SUSY transformations do *not* anticommute (i.e. $\{s_1, s_2\} \neq 0$). This should be contrasted with the nilpotent (anti-)BRST symmetry transformations which absolutely anticommute (see, e.g. [9-13] for details). Within the framework of superfield approach to (anti-)BRST symmetries, the superfields are expanded along both the Grassmannian directions $(\theta, \bar{\theta})$ (see, e.g. Appendix A).

[‡]We observe that, in the limit $\bar{\theta} = 0$, we get back the *variables* $x(t), y(t), \psi(t)$ and $\bar{\psi}(t)$ from (10). This is why we have christened our present technique as the *supervariable approach* to the description of some $\mathcal{N} = 2$ SUSY quantum mechanical models.

As per prescription laid down in [14,15], we have the following SUSY invariant restrictions (SUSYIRs) on the (super)variables:

$$\begin{aligned}
X(t, \bar{\theta}) \Psi(t, \bar{\theta}) &= x(t) \psi(t), & Y(t, \bar{\theta}) \Psi(t, \bar{\theta}) &= y(t) \psi(t), \\
\dot{X}(t, \bar{\theta}) \dot{\Psi}(t, \bar{\theta}) &= \dot{x}(t) \dot{\psi}(t), & \dot{Y}(t, \bar{\theta}) \dot{\Psi}(t, \bar{\theta}) &= \dot{y}(t) \dot{\psi}(t), \\
\frac{1}{2} \left[\dot{X}^2(t, \bar{\theta}) + \dot{Y}^2(t, \bar{\theta}) \right] + i \bar{\Psi}(t, \bar{\theta}) \dot{\Psi}(t, \bar{\theta}) &= \frac{1}{2} \left[\dot{x}^2(t) + \dot{y}^2(t) \right] \\
+i \bar{\psi}(t) \dot{\psi}(t), & & \Psi(t, \bar{\theta}) &= \psi(t),
\end{aligned} \tag{12}$$

which lead to the following relationships amongst the secondary variables (b_1, b_2, f_1, f_2) of the expansions (10) and the basic variables $(x, y, \psi, \bar{\psi})$, namely;

$$\begin{aligned}
b_1(t) &= 0, & f_1(t) \psi(t) &= 0, & \dot{f}_1(t) \dot{\psi}(t) &= 0, & f_2(t) \psi(t) &= 0, \\
\dot{f}_2(t) \dot{\psi}(t) &= 0, & \dot{x}(t) \dot{f}_1(t) + \dot{y}(t) \dot{f}_2(t) - b_2(t) \dot{\psi}(t) &= 0.
\end{aligned} \tag{13}$$

The non-trivial solution of the above restrictions are $f_1(t) \propto \psi(t)$ and $f_2(t) \propto \psi(t)$. For the algebraic convenience, however, we choose $f_1(t) = \psi(t)$ and $f_2(t) = -i \psi(t)$. It is evident that if we take the help of these relationships, we obtain $b_2 = \dot{x} - i \dot{y}$ from the last entry of (13).

The explicit substitution of (f_1, f_2, b_1, b_2) into the original expansions (10) leads to the following *final* expansions of the anti-chiral supervariables

$$\begin{aligned}
X^{(1)}(t, \bar{\theta}) &= x(t) + \bar{\theta}(\psi) \equiv x(t) + \bar{\theta}(s_1 x), \\
Y^{(1)}(t, \bar{\theta}) &= y(t) + \bar{\theta}(-i \psi) \equiv y(t) + \bar{\theta}(s_1 y), \\
\Psi^{(1)}(t, \bar{\theta}) &= \psi(t) + \bar{\theta}(0) \equiv \psi(t) + \bar{\theta}(s_1 \psi), \\
\bar{\Psi}^{(1)}(t, \bar{\theta}) &= \bar{\psi}(t) + \bar{\theta}[i(\dot{x} - i \dot{y})] \equiv \bar{\psi}(t) + \bar{\theta}(s_1 \bar{\psi}),
\end{aligned} \tag{14}$$

where the superscript (1), placed on the supervariables, denotes the expansions of the supervariables after the application of the SUSYIRs (12). It is evident now that the following geometrical relationship between the SUSY transformations s_1 and the translational generators $\partial_{\bar{\theta}}$ emerges in an explicit fashion [cf. equation (9)]:

$$\frac{\partial}{\partial \bar{\theta}} [\Omega^{(1)}(t, \theta, \bar{\theta})] |_{\theta=0} = s_1 \Omega(t) \equiv \pm i [\Omega(t), Q]_{\pm}, \tag{15}$$

where $\Omega(t) \equiv x(t), y(t), \psi(t), \bar{\psi}(t)$ is the generic variable of the starting Lagrangian L_0 and $\Omega^{(1)}(t, \theta, \bar{\theta})|_{\theta=0}$ stands for the generic supervariables (14) that have been obtained after application of the SUSYIRs (12). A close and careful look at (15) and (14) explains clearly that we have already obtained the SUSY transformations[§]: $s_1 x = \psi$, $s_1 y = -i \psi$, $s_1 \psi = 0$, $s_1 \bar{\psi} = i(\dot{x} - i \dot{y})$ which are present in (2). Their nilpotency is also clear because of the relationship in (15) which states that $s_1^2 = 0$ and $(\partial_{\bar{\theta}})^2 = 0$ are inter-related.

Let us now focus on the SUSY transformations for A_x and A_y and point out the derivation of $s_1 A_x$ and $s_1 A_y$ within the framework of our supervariable approach. Towards

[§]It will be noted that our supervariable approach allows us to choose the secondary variables as has been done in (14) *modulo a constant factor*. This freedom would be exploited in our Sec. 5 for some specific purpose.

this goal in mind, first of all, we generalize the ordinary potentials $A_x(x, y)$ and $A_y(x, y)$ onto their counterpart anti-chiral supervariables on the anti-chiral super-submanifold ($x \rightarrow X^{(1)}$, $y \rightarrow Y^{(1)}$, $A_x \rightarrow \tilde{A}_x$, $A_y \rightarrow \tilde{A}_y$) as

$$\begin{aligned}
A_x(x, y) &\longrightarrow \tilde{A}_x(X^{(1)}, Y^{(1)}) \equiv \tilde{A}_x(x + \bar{\theta} \psi, y - i \bar{\theta} \psi) \\
&= A_x(x, y) + \bar{\theta} \left[(\partial_x A_x(x, y) - i \partial_y A_x(x, y)) \psi \right] \\
&\equiv A_x(x, y) + \bar{\theta} (s_1 A_x(x, y)), \\
A_y(x, y) &\longrightarrow \tilde{A}_y(X^{(1)}, Y^{(1)}) \equiv \tilde{A}_y(x + \bar{\theta} \psi, y - i \bar{\theta} \psi) \\
&= A_y(x, y) + \bar{\theta} \left[(\partial_x A_y(x, y) - i \partial_y A_y(x, y)) \psi \right] \\
&\equiv A_y(x, y) + \bar{\theta} (s_1 A_y(x, y)).
\end{aligned} \tag{16}$$

We note that we have to use the expansions, obtained in (14), for the derivation of SUSY transformations $s_1 A_x(x, y)$ and $s_1 A_y(x, y)$ which are

$$s_1 A_x = (\partial_x A_x - i \partial_y A_x) \psi, \quad s_1 A_y = (\partial_x A_y - i \partial_y A_y) \psi. \tag{17}$$

It is clear that our above results match with the ones listed in (2).

To derive the other SUSY transformations s_2 , beside the first one (i.e. s_1), we take recourse to the chiral supervariables that are generalization of, first of all, the *simple* dynamical variables $(x(t), y(t), \psi(t), \bar{\psi}(t))$ of the starting Lagrangian L_0 . In other words, we generalize the ordinary SUSY theory onto a (1, 1)-dimensional chiral super-submanifold as [14,15]

$$\begin{aligned}
x(t) &\longrightarrow X(t, \theta, \bar{\theta})|_{\bar{\theta}=0} \equiv X(t, \theta) = x(t) + \theta \bar{f}_1(t), \\
y(t) &\longrightarrow Y(t, \theta, \bar{\theta})|_{\bar{\theta}=0} \equiv Y(t, \theta) = y(t) + \theta \bar{f}_2(t), \\
\psi(t) &\longrightarrow \Psi(t, \theta, \bar{\theta})|_{\bar{\theta}=0} \equiv \Psi(t, \theta) = \psi(t) + i \theta \bar{b}_1(t), \\
\bar{\psi}(t) &\longrightarrow \bar{\Psi}(t, \theta, \bar{\theta})|_{\bar{\theta}=0} \equiv \bar{\Psi}(t, \theta) = \bar{\psi}(t) + i \theta \bar{b}_2(t),
\end{aligned} \tag{18}$$

where (\bar{b}_1, \bar{b}_2) and (\bar{f}_1, \bar{f}_2) are the bosonic and fermionic secondary variables, respectively. It is crystal clear that the bosonic $(x, y, \bar{b}_1, \bar{b}_2)$ and fermionic $(\psi, \bar{\psi}, \bar{f}_1, \bar{f}_2)$ d.o.f. *do* match on the r.h.s. of the expansions (18) which is a key requirement of any arbitrary SUSY theory.

As proposed in the augmented version of superfield formalism [9-13] and in our earlier works [14,15], we have to find out the SUSY invariant quantities under s_2 and demand that they should be independent of the Grassmannian variables θ and $\bar{\theta}$ when they are generalized onto the appropriate supermanifold. In this regards, we note that the following

$$\begin{aligned}
s_2 [\bar{\psi}(t)] &= 0, \quad s_2 [x(t) \bar{\psi}(t)] = 0, \quad s_2 [y(t) \bar{\psi}(t)] = 0, \quad s_2 [\dot{x}(t) \dot{\bar{\psi}}(t)] = 0, \\
s_2 [\dot{y}(t) \dot{\bar{\psi}}(t)] &= 0, \quad s_2 \left[\frac{1}{2} (\dot{x}^2(t) + \dot{y}^2(t)) - i \dot{\bar{\psi}}(t) \psi(t) \right] = 0.
\end{aligned} \tag{19}$$

As a consequence, we have the following interesting and important SUSYIRs on the (su-

per)variables, namely;

$$\begin{aligned}
X(t, \theta) \bar{\Psi}(t, \theta) &= x(t) \bar{\psi}(t), & Y(t, \theta) \bar{\Psi}(t, \theta) &= y(t) \bar{\psi}(t), \\
\dot{X}(t, \theta) \dot{\bar{\Psi}}(t, \theta) &= \dot{x}(t) \dot{\bar{\psi}}(t), & \dot{Y}(t, \theta) \dot{\bar{\Psi}}(t, \theta) &= \dot{y}(t) \dot{\bar{\psi}}(t), \\
\frac{1}{2} [\dot{X}^2(t, \theta) + \dot{Y}^2(t, \theta)] - i \dot{\bar{\Psi}}(t, \theta) \Psi(t, \theta) &= \frac{1}{2} [\dot{x}^2(t) + \dot{y}^2(t)] \\
-i \dot{\bar{\psi}}(t) \psi(t), & & \bar{\Psi}(t, \theta) &= \bar{\psi}(t).
\end{aligned} \tag{20}$$

Using the expansions from (18), we obtain the following:

$$\begin{aligned}
\bar{b}_1(t) &= 0, & \bar{f}_1(t) \bar{\psi}(t) &= 0, & \dot{\bar{f}}_1(t) \dot{\bar{\psi}}(t) &= 0, & \bar{f}_2(t) \bar{\psi}(t) &= 0, \\
\dot{\bar{f}}_2(t) \dot{\bar{\psi}}(t) &= 0, & \dot{x}(t) \dot{\bar{f}}_1(t) + \dot{y}(t) \dot{\bar{f}}_2(t) - \bar{b}_1(t) \dot{\bar{\psi}}(t) &= 0.
\end{aligned} \tag{21}$$

The non-trivial solution of the above restrictions are $\bar{f}_1(t) \propto \bar{\psi}(t)$ and $\bar{f}_2(t) \propto \bar{\psi}(t)$. For the algebraic convenience, however, we choose $\bar{f}_1(t) = \bar{\psi}(t)$ and $\bar{f}_2(t) = i \bar{\psi}(t)$. Using these values (i.e. $\bar{f}_1 = \bar{\psi}$, $\bar{f}_2 = i \bar{\psi}$), we obtain $\bar{b}_1 = \dot{x} + i \dot{y}$. The substitution of the above secondary variables in the equation (18) of the supervariable expansions leads to

$$\begin{aligned}
X^{(2)}(t, \theta) &= x(t) + \theta (\bar{\psi}) \equiv x(t) + \theta (s_2 x), \\
Y^{(2)}(t, \theta) &= y(t) + \theta (i \bar{\psi}) \equiv y(t) + \theta (s_2 y), \\
\bar{\Psi}^{(2)}(t, \theta) &= \bar{\psi}(t) + \theta (0) \equiv \bar{\psi}(t) + \theta (s_2 \bar{\psi}), \\
\Psi^{(2)}(t, \theta) &= \psi(t) + \theta [i (\dot{x} + i \dot{y})] \equiv \psi(t) + \theta (s_2 \psi),
\end{aligned} \tag{22}$$

where the superscript (2) stands for the expansions obtained after the application of SUSYIRs (20). It is clear, from the above expansions, that we have already derived the nilpotent transformations s_2 (cf. (2)).

We derive the SUSY transformations s_2 for the potential functions $A_x(x, y)$ and $A_y(x, y)$. First of all, we generalize these ordinary variables onto the (1, 1)-dimensional chiral super-submanifold as follows [cf. (22)]:

$$\begin{aligned}
A_x(x, y) &\longrightarrow \tilde{A}_x(X^{(2)}, Y^{(2)}) \equiv \tilde{A}_x(x + \theta \bar{\psi}, y + i \theta \bar{\psi}) \\
&= A_x(x, y) + \theta \left[\bar{\psi} (\partial_x A_x(x, y) + i \partial_y A_x(x, y)) \right] \\
&\equiv A_x(x, y) + \theta (s_2 A_x(x, y)), \\
A_y(x, y) &\longrightarrow \tilde{A}_y(X^{(2)}, Y^{(2)}) \equiv \tilde{A}_y(x + \theta \bar{\psi}, y + i \theta \bar{\psi}) \\
&= A_y(x, y) + \theta \left[\bar{\psi} (\partial_x A_y(x, y) + i \partial_y A_y(x, y)) \right] \\
&\equiv A_y(x, y) + \theta (s_2 A_y(x, y)).
\end{aligned} \tag{23}$$

A careful observation at the expansions (22) and (23) demonstrates that we have already obtained the SUSY transformations s_2 [cf. (2)] for all the relevant variables of the theory[¶]. We further note that the following mappings do exist, namely;

$$\frac{\partial}{\partial \theta} [\Sigma^{(2)}(t, \theta, \bar{\theta})] |_{\bar{\theta}=0} = s_2 \Sigma(t) \equiv \pm i [\Sigma(t), \bar{Q}]_{\pm}, \tag{24}$$

[¶]We would like to emphasize that all our transformations can be modified by a constant factor without violating the sanctity of our method. We have used such kind of modifications in our Sec. 5 for some specific purposes.

where $\Sigma(t)$ is the generic variable of the Lagrangian (1) [i.e. $\Sigma(t) = x(t), y(t), \psi(t), \bar{\psi}(t), A_x(x, y), A_y(x, y)$] and $\Sigma^{(2)}(t, \theta, \bar{\theta})|_{\bar{\theta}=0}$ denotes the supervariables [cf. (22), (23)] that have been obtained after the application of the SUSYIRs (20). We note (from (24)) that the nilpotent symmetry transformations s_2 and corresponding charge \bar{Q} are intimately related to the translational generator $\partial_{\bar{\theta}}$ along the Grassmannian direction of the chiral super-submanifold.

4 Invariance of Lagrangian and nilpotency of super-charges: Geometrical supervariable approach

As far as the invariance of the Lagrangian L_0 of (1), under the SUSY symmetry transformations s_1 is concerned, we observe that the starting Lagrangian L_0 can be generalized onto a (1, 1)-dimensional *anti-chiral* supermanifold in the following manner:

$$\begin{aligned} L_0 \Rightarrow \tilde{L}_0^{(ac)} &= \frac{1}{2} \left[\dot{X}^{(1)}(t, \bar{\theta}) \dot{X}^{(1)}(t, \bar{\theta}) + \dot{Y}^{(1)}(t, \bar{\theta}) \dot{Y}^{(1)}(t, \bar{\theta}) \right] \\ &- \left[\dot{X}^{(1)}(t, \bar{\theta}) \tilde{A}_x(X^{(1)}, Y^{(1)}) + \dot{Y}^{(1)}(t, \bar{\theta}) \tilde{A}_y(X^{(1)}, Y^{(1)}) \right] \\ &+ \left[\partial_x \left(\tilde{A}_y(X^{(1)}, Y^{(1)}) \right) - \partial_y \left(\tilde{A}_x(X^{(1)}, Y^{(1)}) \right) \right] \bar{\Psi}^{(1)}(t, \bar{\theta}) \Psi^{(1)}(t, \bar{\theta}) \\ &+ i \bar{\Psi}^{(1)}(t, \bar{\theta}) \dot{\Psi}^{(1)}(t, \bar{\theta}), \end{aligned} \quad (25)$$

where all the supervariables, present in the Lagrangian $\tilde{L}_0^{(ac)}$, are the *ones* that have been derived in (14) as well as (16) and the superscript *(ac)* stands for the anti-chiral behavior of the Lagrangian $\tilde{L}_0^{(ac)}$. In view of the mapping (15), the invariance of the starting Lagrangian L_0 under s_1 can be captured within the framework of the supervariable approach as:

$$\frac{\partial}{\partial \bar{\theta}} \tilde{L}_0^{(ac)} = - \frac{d}{dt} \left[(A_x - i A_y) \psi \right] \Longleftrightarrow s_1 L_0 = - \frac{d}{dt} \left[(A_x - i A_y) \psi \right]. \quad (26)$$

The above equation encapsulates the geometrical meaning of the invariance of the starting Lagrangian L_0 . This can be stated in the language of the translation along the Grassmannian direction $\bar{\theta}$. In fact, the above equation (26) demonstrates that the SUSY Lagrangian $\tilde{L}_0^{(ac)}$ of the theory is a sum of composite supervariables such that its translation along the Grassmannian $\bar{\theta}$ -direction produces a total time derivative in the ordinary spacetime.

Exactly the above kind of analysis can be performed for the invariance of the starting Lagrangian L_0 under the SUSY transformations s_2 . For instance, it can be checked that the starting Lagrangian L_0 can be generalized, onto the (1, 1)-dimensional *chiral* super-submanifold, as

$$\begin{aligned} L_0 \Rightarrow \tilde{L}_0^{(c)} &= \frac{1}{2} \left[\dot{X}^{(2)}(t, \theta) \dot{X}^{(2)}(t, \theta) + \dot{Y}^{(2)}(t, \theta) \dot{Y}^{(2)}(t, \theta) \right] \\ &- \left[\dot{X}^{(2)}(t, \theta) \tilde{A}_x(X^{(2)}, Y^{(2)}) + \dot{Y}^{(2)}(t, \theta) \tilde{A}_y(X^{(2)}, Y^{(2)}) \right] \\ &+ \left[\partial_x \left(\tilde{A}_y(X^{(2)}, Y^{(2)}) \right) - \partial_y \left(\tilde{A}_x(X^{(2)}, Y^{(2)}) \right) \right] \bar{\Psi}^{(2)}(t, \theta) \Psi^{(2)}(t, \theta) \\ &+ i \bar{\Psi}^{(2)}(t, \theta) \dot{\Psi}^{(2)}(t, \theta), \end{aligned} \quad (27)$$

where all the supervariables of $\tilde{L}_0^{(c)}$ owe their origin to the superexpansions (22) and (23) and the superscript (c) on the Lagrangian shows its chiral behavior. In view of the relationship in (24), it is obvious that

$$\begin{aligned} \frac{\partial}{\partial \theta} \tilde{L}_0^{(c)} &= \frac{d}{dt} \left[\bar{\psi} \{ \dot{x} + i \dot{y} - (A_x + i A_y) \} \right] \\ \iff s_2 L_0 &= \frac{d}{dt} \left[\bar{\psi} \{ \dot{x} + i \dot{y} - (A_x + i A_y) \} \right]. \end{aligned} \quad (28)$$

The above relationship provides the geometrical meaning for the invariance of starting Lagrangian L_0 in the ordinary space under s_2 .

Now we concentrate on the geometrical interpretation for the nilpotency of the supercharges $Q(\bar{Q})$ in the language of the translational generators $(\partial_\theta, \partial_{\bar{\theta}})$ along the $(\theta, \bar{\theta})$ directions of the (1, 1)-dimensional super-submanifolds of the general (1, 2)-dimensional supermanifold. Towards this goal in mind, we note that we can express the supercharge Q in terms of the anti-chiral supervariables, in *three* different ways as:

$$\begin{aligned} Q &= \frac{\partial}{\partial \bar{\theta}} \left[-i \bar{\Psi}^{(1)}(t, \bar{\theta}) \Psi^{(1)}(t, \bar{\theta}) \right] \\ &\equiv \int d\bar{\theta} \left[-i \bar{\Psi}^{(1)}(t, \bar{\theta}) \Psi^{(1)}(t, \bar{\theta}) \right], \\ Q &= \frac{\partial}{\partial \bar{\theta}} \left[(\dot{x}(t) - i \dot{y}(t)) X^{(1)}(t, \bar{\theta}) \right] \\ &\equiv \int d\bar{\theta} \left[(\dot{x}(t) - i \dot{y}(t)) X^{(1)}(t, \bar{\theta}) \right], \\ Q &= \frac{\partial}{\partial \bar{\theta}} \left[i (\dot{x}(t) - i \dot{y}(t)) Y^{(1)}(t, \bar{\theta}) \right] \\ &\equiv \int d\bar{\theta} \left[i (\dot{x}(t) - i \dot{y}(t)) Y^{(1)}(t, \bar{\theta}) \right], \end{aligned} \quad (29)$$

where the ordinary variables are from (1) and the supervariables are from the superexpansions (14) and (16).

In view of the mapping (15), the above charge (Q) can be *also* expressed in the ordinary space as follows:

$$Q = s_1 \left[-i \bar{\psi} \psi \right], \quad Q = s_1 \left[(\dot{x} - i \dot{y}) x \right], \quad Q = s_1 \left[i (\dot{x} - i \dot{y}) y \right]. \quad (30)$$

Now the nilpotency of the charge Q becomes pretty trivial in the sense that it is connected with the nilpotency of the transformations s_1 through the relationship: $s_1 Q = +i \{Q, Q\} = 0$ due to $s_1^2 = 0$. This observation could be also captured in the language of the translational generator $\partial_{\bar{\theta}}$ because we observe that $\partial_{\bar{\theta}} Q = 0$ (due to expressions of Q listed in (29)) where we note that it is the nilpotency of the translational generator $\partial_{\bar{\theta}}$ (i.e. $\partial_{\bar{\theta}}^2 = 0$) which is responsible for the proof of the nilpotency of Q .

We focus on the nilpotency of \bar{Q} in the language of geometry on the chiral super-submanifold. Towards this goal in mind, we can also express the supercharge \bar{Q} in terms of

the chiral supervariables, obtained after the application of SUSYIRs (20), in *three* different ways as:

$$\begin{aligned}
\bar{Q} &= \frac{\partial}{\partial \theta} \left[i \bar{\Psi}^{(2)}(t, \theta) \Psi^{(2)}(t, \theta) \right] \\
&\equiv \int d\theta \left[i \bar{\Psi}^{(2)}(t, \theta) \Psi^{(2)}(t, \theta) \right], \\
\bar{Q} &= \frac{\partial}{\partial \theta} \left[X^{(2)}(t, \theta) \left(\dot{x}(t) + i \dot{y}(t) \right) \right] \\
&\equiv \int d\theta \left[X^{(2)}(t, \theta) \left(\dot{x}(t) + i \dot{y}(t) \right) \right], \\
\bar{Q} &= \frac{\partial}{\partial \theta} \left[-i Y^{(2)}(t, \theta) \left(\dot{x}(t) + i \dot{y}(t) \right) \right] \\
&\equiv \int d\theta \left[-i Y^{(2)}(t, \theta) \left(\dot{x}(t) + i \dot{y}(t) \right) \right], \tag{31}
\end{aligned}$$

where the ordinary variables are from (1) and the supervariables are from the expansions (22) and (23). In view of the mapping (24), we can express (31) in the ordinary space as:

$$\bar{Q} = s_2 \left[i \bar{\psi} \psi \right], \quad \bar{Q} = s_2 \left[x (\dot{x} + i \dot{y}) \right], \quad \bar{Q} = s_2 \left[-i y (\dot{x} + i \dot{y}) \right]. \tag{32}$$

The above two equations (31) and (32) show that the charge \bar{Q} can be expressed in terms of nilpotent ($s_2^2 = 0$) transformations s_2 and nilpotent ($\partial_\theta^2 = 0$) translational generator (∂_θ).

A close look at (31) and (32) clarify the nilpotency of the charge \bar{Q} which is beautifully intertwined with the nilpotency of s_2 (i.e. $s_2^2 = 0$) and/or nilpotency ($\partial_\theta^2 = 0$) of the translational generator ∂_θ on the chiral super-submanifold. This can be verified by the observation that $s_2 \bar{Q} = +i \{ \bar{Q}, \bar{Q} \} = 0$ due to nilpotency of s_2 . Similarly, we note that $\partial_\theta \bar{Q} = 0$ because of the nilpotency ($\partial_\theta^2 = 0$) of the generator ∂_θ .

5 Cohomological aspects: Continuous $\mathcal{N} = 2$ SUSY symmetries

For the sake of completeness of our paper, we concisely point out the mathematical meaning of the symmetry transformation operators (s_1, s_2, s_ω) that have been mentioned in equations (2) and (4). Towards this goal in mind, we modify the transformations (2) by a constant

factor^{||} as [17]

$$\begin{aligned}
s_1 x &= \frac{\psi}{\sqrt{2}}, & s_1 y &= \frac{-i\psi}{\sqrt{2}}, & s_1 \bar{\psi} &= \frac{i}{\sqrt{2}} [\dot{x} - i\dot{y}], & s_1 \psi &= 0, \\
s_1 A_x &= \frac{1}{\sqrt{2}} (\partial_x A_x - i\partial_y A_x) \psi, & s_1 A_y &= \frac{1}{\sqrt{2}} (\partial_x A_y - i\partial_y A_y) \psi, \\
s_2 x &= \frac{\bar{\psi}}{\sqrt{2}}, & s_2 y &= \frac{i\bar{\psi}}{\sqrt{2}}, & s_2 \psi &= \frac{i}{\sqrt{2}} [\dot{x} + i\dot{y}], & s_2 \bar{\psi} &= 0, \\
s_2 A_x &= \frac{\bar{\psi}}{\sqrt{2}} (\partial_x A_x + i\partial_y A_x), & s_2 A_y &= \frac{\bar{\psi}}{\sqrt{2}} (\partial_x A_y + i\partial_y A_y).
\end{aligned} \tag{33}$$

It is straightforward to check that the algebra obeyed by the transformation operators (s_1, s_2, s_ω) is^{**}:

$$\begin{aligned}
s_1^2 &= 0, & s_2^2 &= 0, & \{s_1, s_2\} &= s_\omega = (s_1 + s_2)^2, \\
[s_\omega, s_1] &= 0, & [s_\omega, s_2] &= 0, & \{s_1, s_2\} &\neq 0.
\end{aligned} \tag{34}$$

At the algebraic level, the above algebra is exactly like the algebra obeyed by the de Rham cohomological operators (see, e.g. [18-21])

$$\begin{aligned}
d^2 &= 0, & \delta^2 &= 0, & \{d, \delta\} &= \Delta = (d + \delta)^2, \\
[\Delta, d] &= 0, & [\Delta, \delta] &= 0, & \{d, \delta\} &\neq 0.
\end{aligned} \tag{35}$$

In the above, the operators $(\delta)d$ are the (co-)exterior derivatives (with $d^2 = \delta^2 = 0$) and $\Delta = (d + \delta)^2$ is the absolutely *commuting* Laplacian operator.

In the realm of differential geometry, one knows that the (co-)exterior derivatives are connected by the relation $\delta = \pm * d *$ where $(*)$ is the Hodge duality operation on a given compact manifold on which the set (d, δ, Δ) is defined. In our theory, the $(*)$ operation is replaced by a discrete set of *symmetry* transformations:

$$\begin{aligned}
x &\rightarrow \mp x, & \psi &\rightarrow \mp \bar{\psi}, & A_x &\rightarrow \pm A_x, & t &\rightarrow -t, \\
y &\rightarrow \pm y, & \bar{\psi} &\rightarrow \pm \psi, & A_y &\rightarrow \mp A_y, & B_z &\rightarrow B_z,
\end{aligned} \tag{36}$$

under which the Lagrangian (1) remains invariant and we observe that the nilpotent transformations s_2 and s_1 are connected by [17]

$$\begin{aligned}
s_2 \Phi_1 &= + * s_1 * \Phi_1 \Rightarrow s_2 = + * s_1 *, & \Phi_1 &= x, y, A_x, A_y, \\
s_2 \Phi_2 &= - * s_1 * \Phi_2 \Rightarrow s_2 = - * s_1 *, & \Phi_2 &= \psi, \bar{\psi},
\end{aligned} \tag{37}$$

where (\pm) signs, in the above relationship, are governed by the application of two consecutive discrete symmetry transformations, namely;

$$\begin{aligned}
* [*] \Phi_1 &= + \Phi_1, & \Phi_1 &= x, y, A_x, A_y, \\
* [*] \Phi_2 &= - \Phi_2, & \Phi_2 &= \psi, \bar{\psi}.
\end{aligned} \tag{38}$$

^{||}We have taken a factor of $(1/\sqrt{2})$ in the overall transformations so that the corresponding charges would be able to satisfy one of the simplest form of the $sl(1/1)$ algebra of $\mathcal{N} = 2$ SUSY quantum mechanics (cf. (40) below) where there is no central extension.

^{**}It is elementary to note that the transformations (4) (i.e. s_ω) would be now expressed modulo an i factor because of the modifications in (33) *vis-à-vis* (2).

The above is the rule (for signatures in (37)) laid down by the requirements of a perfect *duality* invariant theory [22]. We note, from equation (38), that it is the interplay of continuous and discrete symmetry transformations of our theory which provide the physical realization of the relationship between the (co-)exterior derivatives: $\delta = \pm * d *$ of differential geometry.

For our model under consideration, we note that the continuous symmetry transformations (s_1, s_2, s_ω) lead to the following expressions for the Noether conserved charges Q_i (with $i = 1, 2, 3$), namely;

$$\begin{aligned} Q_1 &\equiv Q = \frac{1}{2} \left[(p_x + A_x) - i (p_y + A_y) \right] \psi, \\ Q_2 &\equiv \bar{Q} = \frac{\bar{\psi}}{2} \left[(p_x + A_x) + i (p_y + A_y) \right], \\ Q_3 &\equiv Q_\omega = \left[\frac{(p_x + A_x)^2}{2} + \frac{(p_y + A_y)^2}{2} - B_z \bar{\psi} \psi \right] \equiv H. \end{aligned} \quad (39)$$

It can be readily checked that the above charges obey one of the simplest $\mathcal{N} = 2$ SUSY quantum mechanical algebra, namely;

$$\begin{aligned} Q^2 &= 0 & \bar{Q}^2 &= 0, & \{Q, \bar{Q}\} &= H, \\ \dot{Q} &= -i [Q, H] = 0, & \dot{\bar{Q}} &= -i [\bar{Q}, H] = 0, \end{aligned} \quad (40)$$

which provides the physical realization of the Hodge algebra.

6 Conclusions

In our present endeavor, we have taken an example of the $\mathcal{N} = 2$ SUSY quantum mechanical model whose superpotential is totally different from the cases of $\mathcal{N} = 2$ SUSY free particle and HO (and the generalization of HO) [14,15,17]. This has been done *purposefully* so that our idea of the supervariable approach [14,15] could be put on a solid foundation. We have derived the proper $\mathcal{N} = 2$ transformations for the SUSY system under consideration by exploiting the idea of SUSYIRs. We have also provided the geometrical basis for the nilpotency of SUSY transformations and SUSY invariance of the Lagrangian in the language of translational generators $(\partial_\theta, \partial_{\bar{\theta}})$ on the $(1, 1)$ -dimensional chiral and anti-chiral super-submanifolds.

We have demonstrated, in our present investigation, that the nilpotency of a SUSY transformation of an ordinary dynamical variable (of the starting Lagrangian (1)) is intimately connected with a set of two successive translations of the corresponding supervariable along $(\bar{\theta})\theta$ directions of the $(1, 1)$ -dimensional (anti-)chiral super-submanifolds of the general $(1, 2)$ -dimensional supermanifold on which our starting theory is generalized (cf. Sec. 4). Similarly, we have established that the SUSY invariance of the Lagrangian (1) is equivalent to the translation of a sum of composite supervariables (that are present in the (anti-)chiral Lagrangians) along the Grassmannian $(\bar{\theta})\theta$ directions of the (anti-)chiral super-submanifolds such that this process yields a total time derivative in the ordinary space.

Our present work and earlier works [14,15] are our modest *first* few steps towards our main goal of deriving the SUSY transformations with the minimal knowledge about the *classical* Lagrangian and its symmetries. Such expectations and intuitions have been spurred due to our experiences in the application of superfield formalism [4,5,9-13] to the gauge systems. In fact, in the realm of BRST formalism, if one knows the (anti-)BRST symmetries, there is absolutely *no* problem in obtaining the gauge-fixing and Faddeev-Popov ghost terms (see, e.g. [9-13]). Our central ideology is to develop theoretical tools and techniques so that we could derive the whole structure of the SUSY invariant Lagrangian from the knowledge of SUSY symmetry transformations that emerge from the supervariable approach.

So far, we have applied our supervariable approach to the derivation of $\mathcal{N} = 2$ SUSY symmetries for some explicit examples, viz., $\mathcal{N} = 2$ SUSY free particle and HO. Our main goal is to apply the augmented version of superfield approach [9-13] to the SUSY gauge theories that have become important because of their relevance to the modern developments in superstring theories. In fact, our aim is to study the $\mathcal{N} = 2, 4$ and 8 SUSY gauge theories within the framework of BRST formalism where, we are sure, our augmented version of BT-superfield formalism [9-13] would play very important role. As far as SUSY gauge theories are concerned, we have already taken the *first* modest step and supersymmetrized the HC in the context of (SUSY system of) a free spinning relativistic particle and obtained the proper (i.e. nilpotent and absolutely anticommuting) (anti-)BRST transformations [23]. Presently, one of us is also involved with the application of super-HC to derive the (anti-)BRST symmetries in the context of Abelian SUSY gauge theory [24].

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Appendix A: Choice of the (anti-)chiral supervariables for the description of our SUSY model

As we have mentioned in the main body of our text, one of the key differences between the $\mathcal{N} = 2$ SUSY transformations and (anti-)BRST symmetry transformations is the fact that whereas the latter are absolutely anticommuting, the former are *not*. Thus, within the framework of BT-superfield approach to (anti-)BRST symmetry transformations, a generic superfield (defined on a $(D, 2)$ -dimensional supermanifold) is expanded along both the Grassmannian directions $(\theta$ and $\bar{\theta})$ of the supermanifold, namely;

$$\Sigma(x, \theta, \bar{\theta}) = \sigma(x) + \theta \bar{R}(x) + \bar{\theta} R(x) + i \theta \bar{\theta} S(x), \quad (A.1)$$

where $\sigma(x)$ is an ordinary D -dimensional field of the original (anti-)BRST invariant theory and $\Sigma(x, \theta, \bar{\theta})$ is the corresponding superfield on the $(D, 2)$ -dimensional supermanifold that is characterized by the superspace coordinates $Z^M = (x^\mu, \theta, \bar{\theta})$ (with $\theta^2 = \bar{\theta}^2 = 0$, $\theta \bar{\theta} + \bar{\theta} \theta = 0$).

It is evident, from (A.1), that *if* $\sigma(x)$ is a bosonic ordinary field, then, $\Sigma(x, \theta, \bar{\theta})$ would be also bosonic (i.e. secondary fields (R, \bar{R}) would be fermionic and $S(x)$ bosonic). On the contrary, if $\sigma(x)$ is fermionic, then, $\Sigma(x, \theta, \bar{\theta})$ and $S(x)$ would be fermionic, too. The pair (R, \bar{R}) would become bosonic in the case of $\sigma(x)$ being fermionic. A natural consequence of the expansion in (A.1) is the observation that

$$\begin{aligned} \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \left(\Sigma(x, \theta, \bar{\theta}) \right) &= i S(x) \iff s_b s_{ab} \sigma(x), \\ \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \left(\Sigma(x, \theta, \bar{\theta}) \right) &= -i S(x) \iff s_{ab} s_b \sigma(x), \end{aligned} \quad (A.2)$$

where $s_{(a)b}$ are the (anti-)BRST symmetries and they are identified with the translational generators $(\partial_\theta)\partial_{\bar{\theta}}$ along the Grassmannian direction $(\theta)\bar{\theta}$ of the $(D, 2)$ -dimensional supermanifold [4-13]. It is clear, from (A.2), that

$$(\partial_{\bar{\theta}} \partial_\theta + \partial_\theta \partial_{\bar{\theta}}) \Sigma(x, \theta, \bar{\theta}) = 0 \iff (s_b s_{ab} + s_{ab} s_b) \sigma(x) = 0, \quad (A.3)$$

which establishes the absolute anticommutativity of the (anti-)BRST symmetry transformations. Furthermore, it is also obvious that the nilpotency (i.e. $\partial_\theta^2 = \partial_{\bar{\theta}}^2 = 0$) of the above translational generators $(\partial_\theta)\partial_{\bar{\theta}}$ implies the off-shell nilpotency of the fermionic (anti-)BRST transformations (i.e. $s_{(a)b}^2 = 0$). Thus, whenever we consider the full expansions (like (A.1)) of the superfield, the nilpotency and absolute anticommutativity properties are automatically implied within the framework of superfield formalism (see, e.g. [4-13]).

The application of our supervariable approach to a SUSY system theoretically compels us to choose the (anti-)chiral supervariables so that we could capture *only* the nilpotency property but *avoid* the absolute anticommutativity of the $\mathcal{N} = 2$ SUSY transformations. The latter property is a decisive feature of the $\mathcal{N} = 2$ SUSY quantum mechanical theory.

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